UNITED NATIONS
ECONOMIC AND SOCIAL COUNCIL

Distr.
LIMITED
E/CN.3/L.47
7 April 1958

ORIGINAL: ENGLISH

STATISTICAL COMMISSION
Tenth session
Item 8c of the provisional agenda

A PRELIMINARY ANALYSIS OF THE NATURE OF AGGREGATES AT CONSTANT PRICES

(Memorandum by the Secretary-General)

1. A useful tool of modern economic analysis is the valuation, at constant prices, of aggregates originally valued at current prices. Recent literature on the subject indicates that there are considerable differences between the meanings given by different authors to the concept "aggregate at constant prices". On the one hand, revaluation at constant prices may be directed towards measuring volume changes between the base and the current periods; on the other, it may be directed to studying the effect on the aggregate in current prices of dividing it by certain price indexes. Revaluation to measure volume changes, which is typefied by the so called "revaluation of commodity flows", associates with each individual commodity transaction in the current period an adequately similar commodity transaction in the base period and obtains the revalued aggregate by multiplying each individual quantity transacted in the current period by the price at which the similar transaction in the base period took place. The best example of this sort of revaluation is perhaps provided by external trade statistics because they record the quantities, q, and values, v, at which are transacted individual commodities from a very detailed list, thus making it possible to associate with the value, $v_n = p_n q_n$, in current prices (where $p_n = v_n/q_n$ and the subscript n refers to the current period) a value $p_o q_n$ in base period prices. If $\sum$ represents a sum over a number of transactions in the current period, then $\sum p_n q_n$ is revalued as $\sum p_o q_n$. This method of revaluing is basic to computing the Laspeyres quantum index,

$Q = \sum p_o q_n / \sum p_o q_o$, and the Paasche price (or unit value) index, $P = \sum p_n q_n / \sum p_o q_n$. The aggregate in current prices can thus be revalued by dividing it by the price index P, for in the case of commodity flows

(1) $$(\sum v_n)/P = \sum p_o q_n$$

2. Certain transactions other than commodity flows, such as for instance wage payments, can, conceptually at least, be revalued in much the same way as commodity flows; the number, q_n, of man-hours worked in each occupation in the current period could, for instance, be multiplied by the wage rate, p_o, in the base period for that occupation to give aggregate wage payments in constant prices. Similarly profits, interest, rent, taxes, etc. could be revalued in detail by comparing, for each individual transaction, the rates applying in the base and current periods; a formula like (1) also holds when revaluation is of this sort. It may happen that the same aggregate in current prices can be revalued on the one hand from the output side as a sum of commodity flows and on the other from the input side as a sum of wages, profits, interest, etc. The difference, if any, between two aggregates in constant prices which result from two different revaluations of the same aggregate in current prices will here be provisionally termed the "Deflation Defect". It will be indicated that, if the revaluations are by input and output respectively, in the ways just described, the deflation defect will be zero if there has been no technical change between the base and current periods and, if there has been technical change, will be the algebraic sum of the effects of technical change on each individual transaction. Technical change here means a change in the proportions borne by inputs (including factors) in total output, all expressed in constant prices; it is algebraically defined in Annex B to this paper. A major constituent in what has been called the deflation defect will be seen to be changes between the base and current periods in productivity. The result of this analysis makes it possible to estimate the differences to be expected between revaluing gross domestic product at market prices,

Y, from the output side on the one hand and from the input side on the other and to study the effect on revaluation of subtracting the input M from the output C + F + E in the expression

$$Y = C + F + E - M$$

where C = private and general government consumption, F = gross domestic capital formation, E = exports of goods and services, M = imports of goods and services.

3. When P' is a price index obtained by some other method than the association, described in paragraphs 1 and 2, of individual transactions in the current period with similar transactions in the base period, the expression

(2) $$(\sum v_n)/P'$$

is frequently also called an aggregate at constant prices. The following choices of P', different from the P of equation (1), have recently been made in various contexts to revalue the trade balance E - M:

(a) P' = import unit value index[1/]

(b) P' = export unit value index[2/]

(c) P' = wholesale price index[3/]

Because the expression (2) does not satisfy the relation (1) the results of these procedures will, in general, differ from the result of revaluing by comparing individual transactions and they will also usually differ among themselves thus generating deflation defects. Each deflation defect will, in general, have an economic meaning. If, for instance, P_e and P_m are the export and import unit value indexes, then $(E - M)/P_m - (E/P_e - M/P_m) = E(1/P_m - 1/P_e)$

1/ See, for instance, "Statistics of National Product and Expenditure 1938, 1947-1952" p.128, OEEC, Paris, 1954.

2/ See, for instance, "A System of Price and Quantity Indexes for National Accounts" United Nations document E/CN.3/231 and "A System of Price and Quantity Indexes for National Accounts" United Nations document E/CN.3/L.46, para.14.

3/ See, for instance, paragraph 3 of document E/CN.3/L.46 cited in footnote 2.

is the deflation defect to which (a) above gives rise when compared to revaluation by the commodity flows method. In this case the deflation defect measures one of the many economic effects of changes in the terms of trade, namely the gain, between the two periods, in the imports that actual exports would finance. In chapter 2 of document E/CN.3/L.46 and elsewhere, Y is revalued, by the commodity flow method, as output $C + F + E$ less the input M, all at constant prices.

4. In order to study the relation between the results of revaluing inputs and revaluing outputs, it is useful to conceive of every output transaction as analysed into a sum of basic transactions, each of which can be revalued as if it were a commodity flow. Furthermore the basic transactions are so chosen that input aggregates, such as wages, interest, rents, etc., also turn out each to be a sum of basic transactions. After the basic transactions are revalued they can either (a) be re-assembled to make up the various goods and services which constitute output, or (b) be assembled in a different way to make up the aggregates into which inputs are traditionally analysed. If the revaluation of the basic transactions is made as if they were commodity flows, the result of (a) will be the same figure as that obtained by revaluing the original goods and services in the ordinary way by commodity flows, which was discussed in paragraph 1. If the revaluation of the basic transactions is made from the input point of view, the result of (b) will be the figures discussed in paragraph 2. Revaluing a basic transaction as an output, as at (a), may differ from revaluing it as an input, as at (b), thus giving rise to a deflation defect, because the quantity units in which input is measured may differ from those in which output is measured and, if there is technical change, the number of input units per output unit may change between the base and current periods. Thus, for example, if only labour adds value to a commodity, the result of revaluing input (man-hours) may differ, as the example below shows, from the result of revaluing output (number of units produced of the commodity). This difference, which gives rise to a deflation defect, will occur only if there have been technical changes in the production of the commodity, i.e. only when

changes in input quantities do not adequately represent changes in output quantities.

Example 1	Base period	Current period
Value produced of the commodity, $	2000	3600
Input: man-hours worked	200	300
price, wages per man-hour	10	12
Output: number of units produced	400	800
price per unit	5	4.5

Output at constant prices = 800 x \$5 = \$4000
Input at constant prices = 300 x \$10- \$3000

Deflation defect = \$1000

The deflation defect, $(\frac{800}{400} - \frac{300}{200}) \times \$2000$, is proportional to the difference, in brackets, between the two quantity ratios (number of commodity units and number of man-hours) basic to the manufacturing process. Annex B provides a treatment in general terms of this sort of situation. Where possible the deflation defect will in this paper be written so as to represent the excess of output in constant prices over input in constant prices. In this example changes in productivity entirely account for the deflation defect. In what follows other elements which may contribute are identified.

5. Because imports, M, constitute an input and the other terms of the expression C + F + E - M represent outputs, the difference between revalued inputs and revalued outputs has a particularly important effect on the revaluation of C + F + E - M. Each term of M is, in the expression, balanced by a term of C + F + E and the analysis, referred to in paragraph 4, of each transaction into basic components will be made below in such a way that each input transaction of M can be precisely associated with the output component of C + F + E which balances it. If the unit of quantity of the input is the same as that of the balancing output (as, for instance, when an imported pair of shoes is sold to a domestic consumer) no deflation defect is generated. When, on the other hand, the quantity unit of input disappears in the process of producing

output (as when imported tin is absorbed in the manufacture of tin-plated steel sheets) a deflation defect will be generated if, between the base and current periods, quantities of different input components are, as in the following simplified example, subject to different movements.

<u>Example 2</u>

			Base period	Current period
Input:	imported tin:	value, million $	0.8	1.2
		quantity, thou.tons	4	5
		price	0.2	0.24
	steel:	value,million $	1	2
		quantity,million sq. mtrs.	1	1.6
		price	1	1.25
	labour:	value, million $	0.2	0.4
		quantity, thou. m-h	20	35
		price	0.01	0.014
Output:	tin plate	value, million $	2.0	3.6
		quantity, million sq. mtrs.	1	1.6
		price	2	2.25

Output at constant prices (million $) = 1.6 x 2 = 3.2

Input at constant prices (million $) = 5(.2) + 1.6(1) + 35(.01) = 2.95

Deflation defect = 0.25

Part III of Annex B paragraphs 2-5 discusses in general terms the revaluation of imports.

6. It is convenient to denote by A(X) the result of revaluing X by the method which is an extension of revaluation of commodity flows. The value A(E) - A(M) may be negative when E - M is positive, or vice versa. This appears merely to be the arithmetic expression of the fact that transactions giving a positive balance at one set of prices may well give a negative balance at another and to mean that either more or less of C + F + E would be required to balance -M at constant than at current prices. In any case, A(E) - A(M) will, as Annex B shows, differ only by the deflation defect from A(E - M) and this difference will usually be small unless the base and current periods are far apart.

7. The extension to all transactions of the methods used to revalue

commodity flows, depends on the conception that:

(A) the value of each individual commodity or service (output) is suitably subdivided into its components (inputs)

(B) losses are admitted to the system with negative signs.

As is the case for any system for revaluation, statistical and practical limitations prevent the exact application, in any given instance, of the system about to be described. The concept none the less has the virtues of being a guide in practical applications and of making it possible in practice to distinguish deflation defects due to technical changes from deflation defects due to revaluation by methods, such as those referred to in paragraph 3, outside the scheme here being described.

8. In the new system the value of every transaction in every finished commodity or service is conceived of as analysed, in each period, according to the scheme shown immediately below[4/]; the corresponding quantity is, in the case of domestic commodities or services, not analysed.

A. Total value of each individual domestic commodity or service[5/] is analysed into the parts due to :

I. Wages and salaries paid to residents

Sub-classified into types of wages or salaries where these are subject to divergent rate changes during the time interval under consideration

II. Profits (including rent, dividends, interest, etc. paid to residents or government). Sub-classified where necessary

III. Savings of public and private corporations Sub-classified where necessary

4/ Annex A explains the relation of each of the headings of the scheme to "A System of National Accounts and Supporting Tables" United Nations Statistical Office, Studies in Methods, Series F, No. 2, 1953. Similar analyses, applied to values in external trade statistics are made in the paper "Problems of Adapting External Trade Statistics for Special Types of Economic Analysis" (E/CN.3/235) which is being presented to the tenth session of the Statistical Commission.

5/ Articles of the same kind are to be classified separately if their end uses (consumption, capital formation, export) differ.

IV. Taxes (indirect and, for corporations, direct)
Sub-classified where necessary

V. Consumption of fixed capital
Sub-classified where necessary

VI. Imports, sub-classified by individual goods and services

VII. Factor income paid to foreigners

B. Total value and quantity of each individual commodity or service imported is analysed into parts destined for the following end uses :

VIII. Private and general government consumption

IX. Gross domestic capital formation

X. Re-export without further processing

XI. Further processing, subdivided according to the individual commodities or services to which it contributes

Let the most detailed headings resulting from the classification scheme be called "items" and call the set of all items the "classification K". The relation of items in classes VI and XI is discussed in paragraph 10. Conceiving of this classification is similar to conceiving of the existence, in each period, of a very detailed input-output matrix, in which the outputs are commodities and the inputs consist of factor shares, analysed as in I-IV above, and imports in commodity and service detail. With each item of K is associated, in each period, a price (p_o in the base and p_n in the current period) obtained by dividing the value of transactions in the item by the quantity (q_o or q_n) associated with it; in the case of domestic goods and services this is the quantity transacted of the commodity or service to which the item belongs. The aggregate $\sum p_n q_n$ in current prices, where $\sum$ runs over any group of items of K, is then revalued as $A(\sum p_n q_n) = \sum p_o q_n$.

9. It is readily seen that, when applied to aggregates like C, F, E or M, the revaluation is the same as that obtained by revaluing commodity flows in the ordinary way. If, for instance, $\hat{p}$, q represent the price and quantity exported of a given article or service, the subdivision which gives rise to classification K results in an analysis of

$\hat{p}$ into p', p", ... and then $\hat{p}q = (p' + p'' + \ldots)q$. If, on the other hand, p, $\hat{q}$ represent the price and quantity imported of a given article or service, the subdivision results in $p\hat{q} = p(q' + q'' + \ldots)$. Examples concerning the production of wool yarn and the import of coal may make this clearer.

Example 3 Output		Base period	Current period
A. Yarn produced:	value, $	10	20
	quantity, kg.	50	80
	price	0.2	0.25
B. Analysis by input			
Raw wool:	value, $	3	7
	price per kg. of yarn	0.06	0.088
Labour:	value, $	6	10
	price per kg. of yarn	0.12	0.125
Capital consumed:	value, $	2	2
	price per kg. of yarn	0.04	0.025
Profit:	value, $	-1	1
	price per kg. of yarn	-0.02	0.012

A. Yarn revalued as a whole = 80 x $0.2 = $16

B. Yarn revalued as a sum of inputs =

80 x $(0.06 + 0.12 + 0.04 - 0.02) = $16

It may be useful to express in words the economic meaning of a typical term of the revaluation of B; 80 x $0.06 is the amount which the raw material in the yarn produced in the current period would have cost if raw material costs, per kilogram of yarn, had been the same in the current period as they were in the base period. Comparing a sum, over a number of commodities, of terms of this type with the corresponding sum in current prices would measure average change in raw material cost per unit of output. This sum may differ, by a deflation defect, from the result of revaluing raw material in terms of its own quantities, e.g. per kilogram of wool rather than per kilogram of yarn.

Example 4		Base period	Current period
Imports			
A. Coal imported:	value, $	500	400
	quantity, tons	25	16
	price per ton	20	25
B. Analysis by end use			
Households:	value, $	100	50
	quantity, tons	5	2
	price per ton	20	25
Manuf. of gas:	value, $	400	350
	quantity, tons	20	14
	price per ton	20	25

A. Coal revalued as a whole = 16 x $20 = $320

B. Coal revalued as a sum of final uses = (14 + 2) x $20 = $320

10. The classification K obviously has the following properties in each period :

(a) Each term Y,C,F,E,M is an exact sum of the values of items of K.

(b) The part common to any pair of terms is an exact sum of the values of items of K. There are six pairs of terms which have non-empty intersections of this kind, viz :

C and M	F and M	E and M
C and Y	F and Y	E and Y

As explained earlier by means of examples, items in one of the non-empty intersections may be revalued differently as members of one aggregate than as members of the other, thus giving rise to non-zero deflation defects. This is particularly true of those imports which belong simultaneously to classes VI and XI of classification K. The question is examined in Annex B to this paper by calculating the deflation defect

$$D = A(E) - A(M) - A(E - M)$$

and the deflation defect is shown to be proportional to the "average technical change" between periods o and n; i.e. to the average change in the proportions in which quantities of imported goods and services contribute to quantities of final goods and services of E. Similar results obtain if E is replaced in the calculation by C or F. The elements common to Y on the one hand and C, F or E on the other can be studied by considering a typical case, the difference between (a) revaluing wages as pay per man-hour and (b) revaluing wages as contributions by wages to a unit of commodities or services produced. Again the calculation, made in Annex B to this paper, shows that the difference is proportional to technical changes, in this case changes in the productivity of labour. Here, therefore, the deflation defect measures one of the important components of welfare. In both cases, the part of the deflation defect inherent in the concept of deflation itself is an average technical change $\sum \epsilon \, p_o \, q_o$ where ϵ is a difference, due to technical change, between quantity ratios q_n/q_o, and $\sum$ runs over all items of M and Y. As appears in Annex B, the form of each ϵ is such that it may be either positive, zero or negative.

ANNEX A

Classification K in terms of "A System of National Accounts and Supporting Tables" 1/

(Refers to paragraph 8 of the main paper)

1. Accounts 1 and 2 and Table IV of "A System of National Accounts and Supporting Tables" make it possible to analyse Gross Domestic Product at Market Prices into the components listed below. As indicated by the list, each of the four negative items is cancelled by parts of certain positive items. The rest of each positive item is balanced by aggregate values from the items indicated of classification K of paragraph 8 of the paper to which this Annex belongs.

List Analysing Gross Domestic Product at Market Prices

Items of classification K are identified by Roman numerals
Items of the List itself are indicated by small letters of the alphabet

	Positive items	Balanced by value from
a.	Wages and salaries	I,y,z
b.	Pay and allowances of armed forces	I
c.	Income received by households from unincorporated enterprises	II,y,z
d.	Rent, interest, dividends, corporate transfers to households and private non-profit institutions	II,w,x,y,z
e.	Savings of public and private corporations	III,w,x,y,z
f.	Direct taxes on corporations	IV
g.	Government profits and rent	II
h.	Indirect taxes	IV
j.	Consumption of fixed capital	V
r.	Factor income paid to foreigners	VII,w,x,y,z
	Less negative items	
w.	Interest on public debt	d,e,r
x.	Interest on consumer debt	d,e,r
y.	Subsidies	a,c,d,e,r
z.	Factor income from abroad	a,c,d,e,r

1/ United Nations Studies in Methods, Series F, No. 2.

ANNEX B

The Deflation Defects to which Changes in Units of Quantity give Rise

(Refers to paragraph 10 of the main text)

I. Wages: pay per man-hour vs. contributions of wages to goods and services.

1. Let W be the total wage bill in the current period at current prices and let x be an item of classification K contributing to W. Let P and Q refer to wage rates per man-hour and man-hours worked, respectively on item x. Let p and q, referring to wage rate per unit of commodity or service and number of units produced, respectively, be the magnitudes associated with x as an item of K and used in this analytical revaluation. Then the wage bill due to x for the commodity is

$$PQ = pq$$

Let $$\frac{q_n}{q_o} = \frac{Q_n}{Q_o} + \epsilon$$ where ϵ is a technical coefficient measuring the change in productivity per man-hour.

Then $$d = P_o q_n - P_o Q_n = \epsilon P_o q_o$$

and the contribution of x to the deflation defect is $\epsilon P_o q_o$. This result is illustrated by Example 1 of paragraph 4 of the main paper.

II. Other cases in which there is ambiguity as to units.

2. Deflation defects of the type just discussed in connexion with wages can also arise in a number of other ways. Profits, for instance, may be revalued per unit good or service produced, or per unit dollar invested or, in the case of equity shares, per share. A natural way to revalue taxes is in terms of the basis on which they are imposed; thus a tax imposed per unit product would be revalued per unit product and a tax on profits would be revalued per unit of profit. The second of these methods would, and the first would not, give rise to a deflation defect when compared with revaluation by means of commodity

flows. Revaluing consumption of fixed capital per unit of capital invested will produce a deflation defect when compared with revaluation per unit product. Each deflation defect can be calculated by a formula analogous to that of paragraph 1 above. The next section of this annex discusses the deflation defect produced when a unit of goods or services is absorbed in producing another good or service.

III. The result of interchanging the order of subtraction and deflation

3. The deflatiion defect

$$I = A(E) - A(M) - A(E - M)$$

is calculated and shown to be proportional to technical changes between periods o and n. Technical change is defined by equation (3) below.

4. If an item j of E contains no contribution from M, then j enters A(E) and A(E - M) in the same way, does not enter A(M) and contributes zero to the deflation defect. If an item x of M does not contribute to E, it enters -A(M) and A(E - M) in the same way, does not enter A(E) and contributes zero to the deflation defect. If an item x of M is in class X of classification K (i.e. is re-exported without processing), it is an item of class VI; it thus enters A(E) and A(M) in the same way, does not enter A(E - M) and contributes zero to the deflation defect. If an item x of M is in class XI (imported commodity destined for further processing), it has a counterpart in class VI and thus contributes value to an individual domestic commodity or service c of E; this case is treated in the next paragraph.

5. For simplicity it will be assumed that c contains exactly two items, x and y, of class XI. The calculation is entirely analogous when one item or more than two items are common to c and class XI. Let p^c, q^c; p^x, q^x and p^y, q^y represent prices and quantities for c, x and y respectively. Then

$$(1) \qquad p^c = p' + p^m$$

where p^m is the value contributed by imports to a unit of c. Each side of equation (2) then expresses the value contributed to c by imports :

$$(2) \qquad p^m q^c = p^x q^x + p^y q^y$$

Let

(3) $$\frac{q_n^c}{q_o^c} = \frac{q_n^x}{q_o^x} + \epsilon_x = \frac{q_n^y}{q_o^y} - \epsilon_y = r$$

where ϵ_x and ϵ_y measure the technical change. Then, if d is the contribution of c to the deflation defect D,

$$d = (p'_o + p_o^m)q_n^c - (p_o^x q_n^x + p_o^y q_n^y) - p'_o q_n^c$$

$$= r\left\{ p_o^m q_o^c - (p_o^x q_o^x + p_o^y q_o^y)\right\} + \epsilon_x p_o^x q_o^x + \epsilon_y p_o^y q_o^y \text{ by (3)}$$

$$= \epsilon_x p_o^x q_o^x + \epsilon_y p_o^y q_o^y \text{ by (2)}$$

Hence $d = 0$ if $\epsilon_x = \epsilon_y = 0$ and otherwise d is a function of the structural change and of the base period values of the import items considered. The total deflation defect D is the sum of the terms d for all items of class XI.

6. Similar results obtain when E is replaced by C or F.

7. The analysis of paragraphs 2-5 above can be illustrated by carrying on Example 2 of paragraph 5 of the main paper. It will be assumed that the tin plate of the example is to be exported.

Example		Base period	Current period
Import			
	Tin: value, million $	0.8	1.2
q^x	quantity, thou. tons	4	5
p^x	price	0.2	0.24
Export			
	Tin plate: value, million $	2.0	3.6
q^c	quantity, million sq. mtrs.	1.0	1.6
p^c	price	2.0	2.25
Analysis, according to classification K			
p^m	part of p^c due to tin	0.8	0.75
p'	rest of p^c	1.2	1.5

Example, (cont.)

$A(E) \quad = p_o^c q_n^c \quad = 3.2$

$A(M) \quad = p_o^x q_n^x \quad = 1.0$

$A(E - M) \quad = p_o' q_n^c \quad = 1.92$

$$d = A(E) - A(M) - A(E - M) = 0.28$$

$$= \epsilon_x p_o^x q_o^x = \left(\frac{1.6}{1.0} - \frac{5}{4}\right)0.8 = 0.28$$

Example 2 of paragraph 5 takes account not only of the deflation defect due to the imported tin but also of deflation defects due to changes in the productivities of steel (zero) and labour (computed below).

$$\epsilon\, p_o q_o = \left(\frac{1.6}{1.0} - \frac{35}{20}\right)0.2 = -0.03$$

The aggregate deflation defect is thus 0.28 - 0.03 = 0.25 as in example 2.